题目：Calderón inverse problem :From History to a polygonal geometry prior information case

报告人：戴天瑞，中山大学（珠海）

时间：2026年3月9日（星期一）19:30-21:00

地点：东区第五教学楼5107教室

摘要：

Calderón problem which was named after the Argentine mathematician Alberto Pedro Calderón is one of the most import inverse problem in electrical prospection. Rough speaking, one hope to determine the electrical conductivity of a medium by using voltage and current measurements on its boundary. More specifically, given an L^∞ isotropic conductivity γ on an open bounded domain Ω, we hope to study the following map:

$$F : \gamma \mapsto \Lambda_\gamma,$$

where Λ_γ denotes the Dirichlet-to-Neumann map which is defined for a function f on $\partial\Omega$ as a Neumann value:

$$\Lambda_\gamma(f)=\gamma\frac{\partial u}{\partial n},$$

here, n represents the outer normal vector of $\partial\Omega$ and u represents the solution of the following Dirichlet boundary problem:

$$\begin{cases} -\mathbf{div}(\gamma\nabla u) = 0 & \mathbf{in}\ \Omega \\ u = f & \mathbf{on}\ \partial\Omega \end{cases}.$$

Such problem has a long history since the foundational work[1] of Calderón in 1980. It is not until 2006 when, by means of quasi-conformal mappings, K.Astala and L. Paivarinta[2] were able to establish the injectivity of F. Some other planer results and higher dimensional case were contributed by R.Brown,R. Torres,G. Uhlmann, J.Sylvester and so on[2-6]. On the other hand, besides the unique determination study of such D-N map, the stability of this inverse problem has also attracted considerable interest, in other words, whether the map

$$\mathcal{F} : \Lambda_\gamma \mapsto \gamma$$

is continuous. Such continuity highly relies on some prior information or prior assumption made for γ and the situation varies depending on dimensions.

In the first part of this talk, we are going to give a short review of the history of studying the uniqueness and the stability of such inverse problem.

In the second part of this talk, we are going to present a specific result where the conductivity γ has a special form of $\gamma = (k-1)\chi_P + 1$ with P and k are unknown polygonal cavity inside Ω and unknown constant. In this case, we show that the inverse problem is not only uniquely solvable but also has a Lipschitz stability behavior. This is a joint work with Sergio Vessella and Elisa Francini in University of Florence.

Reference

[1]Calderón, A.P., 2006. On an inverse boundary value problem. Computational & Applied Mathematics, 25(2-3), pp.133-138.

[2]Astala, K. and Päivärinta, L., 2006. Calderón's inverse conductivity problem in the plane. Annals of Mathematics, pp.265-299.

[3]Brown, R.M and Uhlmann, G.A., 1997. Uniqueness in the inverse conductivity problem for nonsmooth conductivities in two dimensions. Communications in partial differential equations, 22(5-6), pp.1009-1027.

[4]Nachman, A.I., 1996. Global uniqueness for a two-dimensional inverse boundary value problem. Annals of Mathematics, pp.71-96.

[5]Sylvester, J. and Uhlmann, G., 1986. A uniqueness theorem for an inverse boundary value problem in electrical prospection. Communications on Pure and Applied Mathematics, 39(1), pp.91-112.

[6]Brown, R.M., 1996. Global uniqueness in the impedance-imaging problem for less regular conductivities. SIAM Journal on Mathematical Analysis, 27(4), pp.1049-1056.