

Uniqueness of Ground State Solution

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Abstract

We consider the nonlinear ordinary differential equation

$$u''(r) + \frac{m}{r}u'(r) + f(u(r)) = 0, \quad r > 0,$$

with initial condition $u'(0) = 0$. Positive solutions that decay to zero as $r \rightarrow \infty$ are called *ground states*. This equation arises from radial solutions of the semilinear elliptic problem $-\Delta u + f(u) = 0$ in $\mathbb{R}^n$, where $m = n - 1$. Under natural assumptions on f , the ground state solution is known to be unique. This talk revisits classical uniqueness proofs due to Kwong (1989) for the power-type nonlinearity $f(u) = |u|^{p-1}u - u$ and the subsequent refinements by McLeod (1993) and Tang (2003). This talk incorporates some recent joint works with Prof. Dong Li, Xiaoyi Zhang, Chongchun Zeng.