

The 4th Australia-China joint conference on geometric analysis

(Symposium on geometric analysis and nonlinear PDEs)

	Monday (9 Jun.)			Tuesday (10 Jun.)	Wednesday (11 Jun.)	Thursday (12 Jun.)	Friday (13 Jun.)
	WEST ANHUI UNIVERSITY			JING KAI HALL on the 1 st Floor（经开厅）			
	Opening& Group Photo						
9:00							
9:30 - 10:20	Ben Andrews		9:00 - 9:50	Wenshuai Jiang	Min-Chun Hong	Jingyi Chen	Qing Han
10:20 - 10:50	Break		9:50 - 10:20	Break			
10:50 - 11:40	Huichun Zhang		10:20 - 11:10	Florica Cirstea	Zhichao Wang	Giorgio Poggesi	Jianchun Chu
	Back to Hotel		11:10 - 12:00	Xuezhang Chen	Brian Krummel	Shiping Liu	
12:00 - 13:30	Lunch (BAICUI HALL on the 1 st Floor)柏翠厅						
14:00 - 14:50	J I N G K A I H A L L	Kwok-Kun Kwong	Artem Pulemotov		Discussion on West Anhui University’s Discipline Development (I)	James McCoy	Discussion on West Anhui University’s Discipline Development (II)
14:50 - 15:20		Break		Break			
15:20 - 16:10		Lashi Bandara	Cale Rankin			Tim Buttsworth	
16:10 - 17:00		Xiaoshang Jin	Xin Fu			Haotian Wu	
17:30 - 20:00	Dinner (BAICUI HALL on the 1 st Floor)柏翠厅						

Conference Hotel: Shuguang Bozun Hotel at Lu'an

Talks

Ben Andrews (Australian National University)

Title:

Mean curvature flow of strongly convex submanifolds with one spacelike codimension

Abstract:

I will discuss ongoing work with Qiyu Zhou on the evolution of spacelike submanifolds with one timelike codimension in a pseudo-Riemannian background, mostly in the special case of flat backgrounds. Interesting features of this setting include an avoidance principle, preservation of “strong convexity” (meaning that the second fundamental form takes spacelike values), a non-collapsing theorem, and an analogue of the Huisken/Gage-Hamilton Theorem for evolution of strongly convex submanifolds.

Huichun Zhang (Sun Yat-sen University)

Title:

Optimal boundary gradient estimates for harmonic maps from RCD spaces

Abstract:

In this talk, we introduce some developments for the boundary behaviors of Dirichlet heat kernels and the Gauss-Green formula on metric measure spaces with generalized Ricci curvature, so-called RCD spaces; and then its application to the boundary regularity of harmonic maps from RCD spaces to non-positively curved metric spaces. This is based on a joint work with Xi-Ping Zhu.

Kwok-Kun Kwong (The University of Wollongong)

Title:

Geometric inequalities in space forms via weighted Minkowski integral formulas

Abstract:

In this talk, I will present a series of sharp geometric inequalities for closed hypersurfaces in space forms, derived using a weighted version of Hsiung-Minkowski integral formulas and an integral formula for the deficit in Jensen's inequality. In Euclidean space, these results give sharp upper bounds for the deficit in Alexandrov-Fenchel inequalities. While some of them have been previously proved using inverse curvature flows, our approach offers several advantages: it does not require star-shapedness, often requires weaker convexity conditions, provides more quantitative estimates, and, in some cases, yields exact equality rather than just inequality. This enables analysis of the stability of these inequalities. This is joint work with Yong Wei.

Lashi Bandara (Deakin University)

Title:

Geometry and graphical decompositions of elliptically regular boundary conditions

Abstract:

Non-local elliptically regular boundary conditions appeared in geometry through their crucial role in the Atiyah-Patodi-Singer index theorem on manifolds with boundary. Since then, pseudo-differential methods have been the preferred tool in their analysis and development, with a focus on pseudo-local boundary conditions. Nearly all of this analysis was focused on Dirac-type first-order elliptic differential operators. However, in applications to geometry and topology, it is important to consider more general first-order differential operators and understand non-local boundary conditions beyond those that are pseudo-local.

This talk will feature recent work, where using the bounded holomorphic functional calculus, the Dirac-type assumption has been completely relaxed and all possible boundary conditions understood. Importantly, all elliptically regular boundary conditions have been characterised by a so-called graphical decomposition. This generalises all previous results, including pseudo-local theory. Graphical decompositions are a powerful tool, which can be used to perform continuous deformations of boundary conditions, allowing the invariance of the index to be utilised. An important application of this is the relative index theorem for general first-order elliptic operators.

Xiaoshang Jin (Huazhong University of Science and Technology)

Title:

Essential p-capacity-volume estimates for rotationally symmetric manifolds

Abstract:

In this talk, we will introduce the novel basic volumetric estimates for the p-capacities in rotationally symmetric manifolds. As applications, we obtain the sharp weak (p, q)- imbeddings and the precise lower bounds of the principal frequency of the p-Laplacian. This is a joint work with Prof. Jie Xiao.

Wenshuai Jiang (Zhejiang University)

Title:

Nodal set and Singular set of Elliptic and Parabolic PDEs

Abstract:

In this talk, we consider solution of second order Elliptic PDEs and Parabolic PDEs. In the first part we will recall some results about nodal sets and singular sets to the solutions of Elliptic PDEs. In the second part we will study the behavior of nodal set along the Parabolic PDEs. We will study the Hausdorff measure of the nodal set in each time slice. Furthermore, we also study the volume estimate of the sub-level set in time slice.

Florica Cirstea (The University of Sydney)

Title:

Classification and symmetry of global solutions for nonlinear elliptic equations with mixed reaction terms

Abstract:

In this talk, we describe the set of all positive distributional $C^1(\mathbb{R}^N \setminus \{0\})$ -solutions of elliptic equations with mixed reaction terms of the form
$$\begin{equation} \label{unu1} \mathbb{L}_{\{\rho, \lambda, \tau\}}[u] := \Delta u - (N-2+2\rho) \frac{x \cdot \nabla u}{|x|^2} + \lambda \frac{u^\tau}{|x|^{1-\tau}} - |\nabla u|^{1-\tau} \frac{u}{|x|^{1+\tau}} = |x|^\theta u^q \quad \text{in } \mathbb{R}^N \setminus \{0\}, \end{equation}$$
 where $\rho, \lambda, \theta \in \mathbb{R}$ are arbitrary,

$N \geq 2$, $q > 1$ and $\tau \in [0, 1]$. We show that $\Delta u = 1$ has positive solutions if and only if $f_{\rho, \lambda, \tau}(\beta) > 0$, where $\beta = (\theta + 2)/(q - 1)$ and $f_{\rho, \lambda, \tau}(t) = t \left(t + 2\rho \right) + \lambda |t|^{1-\tau}$ for all $t \in \mathbb{R}$. Under the optimal condition $f_{\rho, \lambda, \tau}(\beta) > 0$, we provide the precise asymptotic behaviour near zero and at infinity for all positive solutions of $\Delta u = 1$, together with corresponding existence results. We find that all such solutions are radially symmetric. New and surprising phenomena arise compared with the findings of Cirstea and Fucianu (2021), where $\rho = (2 - N)/2$ and $\tau = 1$. The latter corresponds to equations with a Hardy potential, extensively investigated by many authors for various choices of ρ , λ and θ . This is joint work with Huyuan Chen and Aleksandar Miladinovic.

Xuezhang Chen (Nanjing University)

Title:

The σ_2 -curvature and boundary mean curvature equation

Abstract:

Let (M^n, g) , $n = 3, 4$ be a smooth compact manifold with totally non-umbilical boundary. Assume that f is a positive smooth function on M and c is a nonnegative smooth function on ∂M . Under certain assumption on nonnegative curvatures of g , we establish that there is a conformal metric whose σ_2 -curvature and boundary mean curvature equal f and c , respectively. Also our example shows that the nonnegative curvature assumption is necessary. Two obstructions to the existence result are well-known to experts in this field: One is local C^2 estimates near generic boundary; the other is to build the blow-up analysis based on local estimates. This is joint work with Wei Wei.

Artem Pulemotov (The University of Queensland)

Title:

The Poisson equation for G_2 -structures

Abstract:

The investigation of G_2 -structures and exceptional holonomy on 7-dimensional manifolds involves the analysis of a nonlinear Laplace-type operator on 3-forms. We will discuss the existence of solutions to the Poisson equation for this operator. Based on joint work with Timothy Buttsworth (The University of New South Wales).

Cale Rankin (Monash University)

Title:

A geometric approach to estimates for optimal transport maps

Abstract:

The goal of this talk is two-fold: I provide a brief introduction to optimal transport, explaining the central problem and the reason for its importance in modern analysis, and I discuss some recent joint work with Simon Brendle, Flavien Léger, and Robert McCann. The joint work concerns regularity of the optimal transport map. Early regularity results due to Delanoë, Caffarelli, and Urbas (separately) were obtained via a priori estimates for Monge – Ampère equations. However these applied only in special cases, and how to deal with general transportation costs remained an important open problem. This problem was resolved with the extension of these a priori estimates in a groundbreaking work by Ma, Trudinger, and Wang. In this talk I show how the original MTW estimates may be realized as estimates for maximal surfaces in pseudo-Riemannian geometry.

Xin Fu (Westlake University)

Title:

Singular Kahler spaces of complex dimension three with Ricci curvature bounded below

Abstract:

Let X be a projective variety of complex dimension 3 with log terminal singularities. We prove that every singular Kahler metric on X with bounded Nash entropy and Ricci curvature bounded below induces a compact RCD space homeomorphic to the projective variety X itself. Moreover, the tangent cone is unique. This is joint work with Bin Guo and Jian Song.

Min-Chun Hong (The University of Queensland)

Title:

The Yang-Mills flow over higher dimensional Riemannian manifolds

Abstract:

We develop a parabolic formulation of the Uhlenbeck gauge fixing theorem and use it to prove the maximal existence of the Yang-Mills flow over compact n -dimensional Riemannian manifolds for $n \geq 4$. Using an idea of Hamilton's on the Ricci flow, we provide new proofs for uniform estimates of higher covariant derivatives of curvature by improving Moser's iteration technique. Furthermore, we investigate the blow-up of the Yang-Mills flow at the maximal existence time and improve an asymptotical result of Hong-Tian.

Zhichao Wang (Fudan University)

Title:

Existence of embedded minimal tori in three-spheres with positive Ricci curvature

Abstract:

In this joint work with Xingzhe Li, we prove the strong Morse inequalities for the area functional in the space of embedded tori and spheres in the three sphere. As a consequence, we prove that in the three dimensional sphere with positive Ricci curvature, there exist at least 4 distinct embedded minimal tori. Suppose in addition that the metric is bumpy, then the three-sphere contains at least 9 distinct embedded minimal tori.

Brian Krummel (University of Melbourne)

Title:

Uniqueness of tangent cones for area-minimizing currents

Abstract:

In his fundamental work in the 1980s, Almgren showed that for a n -dimensional locally area-minimizing rectifiable current T of codimension > 1 , the Hausdorff dimension of the singular set of T is at most $n-2$. In joint work with Neshan Wickramasekera, we use the relatively elementary parts of Almgren's work to show that T has a unique tangent cone at $\mathcal{H}^{n-2}$ -a.e. singular point of T . Using a simplification of Almgren's methods, we also show that the singular set of T is countably $(n-2)$ -rectifiable.

In the talk, we will focus on some of the details of the proof of uniqueness of tangent cones. Our approach consists of decomposing the set of density q singular points of T into the set of branch point $\mathcal{B}$ at which T decays rapidly to a unique tangent plane and the set $\mathcal{S}$ of all remaining density q singular points. At each point $Z_0 \in \mathcal{S}$, we obtain locally uniform estimates indicating that at each density q singular point Z near Z_0 and each sufficiently small scale, T is significantly closer to some non-planar cone than T is to a plane. Using these locally uniform estimates and a new estimate on the uniform distance of T to a union of pair-wise disjoint planes, we can apply a blow-up of L. Simon and Wickramasekera to show that T has a unique tangent plane at $\mathcal{H}^{n-2}$ -a.e. point of $\mathcal{S}$. Along the way, we show that $\mathcal{S}$ is countably $(n-2)$ -rectifiable.

Jingyi Chen (The University of British Columbia)

Title:

Holomorphic disks with boundary on compact Lagrangian surface

Abstract:

Let L be a compact oriented Lagrangian surface in a Kähler surface endowed with a complete Riemannian metric (compatible with the symplectic structure and the complex structure) with bounded sectional curvatures and a positive lower bound on injectivity radius. We show that for every non-trivial class $[\gamma]$ of the fundamental group $\pi_1(L)$ such that γ bounds a topological disk in M , there exists a holomorphic disk whose boundary belongs to L and is freely homotopic to γ on L .

In the almost Kähler setting, especially, the cotangent bundles of compact manifolds, results on non-existence of J -holomorphic disks and existence of minimizers of the partial energies will be discussed if time permits.

Giorgio Poggesi (The University of Adelaide)

Title:

On critical points of some volume-constrained shape optimisation problems: soap bubbles, overdetermined problems, and related questions

Abstract:

The talk will focus on critical points of certain isoperimetric-type problems, or, more precisely, volume-constrained shape optimisation problems.

The celebrated symmetry theorems established by Alexandrov in 1958 and Serrin in 1971 provide characterisations of critical points for the isoperimetric problem associated with the area functional (the classical isoperimetric problem) and the isoperimetric problem associated with the torsional rigidity functional (Saint Venant's problem), respectively.

These theorems inaugurated exceptionally fruitful research directions that continue to play prominent roles in the contemporary research. We will present various recent generalisations of these results and discuss related quantitative stability estimates.

Shiping Liu (University of Science and Technology of China)

Title:

Discrete Ricci curvature and Terwilliger's conjecture

Abstract:

Given a discrete space with local structural restrictions, it is natural to ask how large can such a space be. We will report recent progresses on the study of the Terwilliger's conjecture (JCTB 1983) on finiteness of the so-called amply regular graphs. We discuss an approach by considering the heat diffusion on such graphs. A key concept is a discrete Ricci curvature notion à la Bakry and Emery which is calculated by eigenvalues of local graphs. As a by product, we derive a weak version of a conjecture due to Qiao, Park and Koolen.

James McCoy (The University of Newcastle)

Title:

Isoperimetric inequality for multiply winding curves

Abstract:

We'll discuss some old and new results for the isoperimetric inequality in the plane, for closed curves of positive integer winding number. In particular, I will outline some work in progress with Yong Wei and Glen Wheeler on a new nonlinear fourth order parabolic curvature flow that can be used to prove an isoperimetric inequality for multiply winding curves of sufficiently small oscillation of curvature. Time permitting, I will outline how this flow can also be used to prove a similar result for embedded curves in the sphere and multiply winding curves in hyperbolic space.

Tim Buttsworth (The University of New South Wales)

Title:

Computationally-assisted existence proofs in differential geometry

Abstract:

In this talk, I will outline a general two-step approach that has the potential to be very useful in the construction of many ad hoc examples of special solutions of geometric PDEs. The first step is to obtain a (verifiably) highly-accurate approximate solution metric using numerical techniques, and the second step is to then use perturbative techniques to prove existence of a true solution near the approximate solution. I will report on recent work I have done with Liam Hodgkinson (Melbourne) which successfully uses this approach to prove existence of a new $O(3) \times O(10)$ -invariant Einstein metric on S^{12} . I will then go on to discuss other possible geometric applications.

Haotian Wu (The University of Sydney)

Title:

Asymptotic behavior of unstable perturbations of the Fubini–Study metric in Ricci flow

Abstract:

The Ricci flow can be regarded as a dynamical system on the space of Riemannian metrics. It is important to identify and study its fixed points, which are generalized Einstein metrics known as Ricci solitons. A prominent example of a Ricci soliton is the Fubini–Study metric on complex projective space. Kröncke has shown that the Fubini–Study metric is an unstable generalized stationary solution of Ricci flow. This raises an interesting question: What happens to Ricci flow solutions that start at arbitrarily small but unstable perturbations of the Fubini–Study metric? In a joint work with Garfinkle, Isenberg and Knopf, we carry out numerical simulations which indicate Ricci flow solutions originating at unstable perturbations of the Fubini–Study metric develop local singularities modeled by the FIK shrinking soliton discovered by Feldman, Ilmanen and Knopf.

Qing Han (Notre Dame University)

Title:

The mass-angular momentum inequality for multiple black holes

Abstract:

In this talk, we study the mass-angular momentum inequality for multiple black holes, modulo the extreme black hole ‘no hair theorem’. More precisely we will prove that either there is a counterexample to black hole uniqueness, in the form of a regular axisymmetric stationary vacuum spacetime with an asymptotically flat end and multiple degenerate horizons which is ‘ADM minimizing’, or the following statement holds. Complete, simply connected, maximal initial data sets for the Einstein equations with multiple ends that are either asymptotically flat or asymptotically cylindrical, admit an ADM mass lower bound given by the square root of total angular momentum, under the assumption of nonnegative energy density and axisymmetry. Moreover, equality is achieved in the mass lower bound only for a constant time slice of an extreme Kerr spacetime. The proof is based on a novel flow of singular harmonic maps with hyperbolic plane target, under which the renormalized harmonic map energy is monotonically nonincreasing. Relevant properties of the flow are achieved through a refined asymptotic analysis of solutions to the harmonic map equations and their linearization.

Jianchun Chu (Peking University)

Title:

On Kahler manifolds with non-negative mixed curvature

Abstract:

The mixed curvature comes from a linear combination of the Ricci and holomorphic sectional curvature. In this talk, we will first survey recent progress. Then we discuss a structure theorem for compact Kahler manifolds with non-negative mixed curvature. This is a joint work with Man-Chun Lee and Jintian Zhu.