A Yamabe problem for the quotient between the Q curvature and the scalar curvature

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We consider a new Yamabe problem for the quotient between the Q curvature and the scalar curvature R: *Find a conformal metric g in a given conformal class $[g_0]$ with*

$$Q_g/R_g = const.$$

When the dimension $n \geq 5$, it relates to a new Sobolev inequality between the total Q-curvature and the total scalar curvature on $\mathbb{S}^n$ ($n \geq 5$), namely

$$\frac{\int_{\mathbb{S}^n} Q_g dv_g}{(\int_{\mathbb{S}^n} R_g dv_g)^{\frac{n-4}{n-2}}} \geq \frac{\int_{\mathbb{S}^n} Q_{g_{\mathbb{S}^n}} dv(g_{\mathbb{S}^n})}{(\int_{\mathbb{S}^n} R_{g_{\mathbb{S}^n}} dv(g_{\mathbb{S}^n}))^{\frac{n-4}{n-2}}},$$

for any g in the conformal class of the round metric $g_{\mathbb{S}^n}$ with positive scalar curvature, with equality if and only if g is also a metric with constant sectional curvature. With this inequality we introduce a new Yamabe constant $Y_{4,2}(M, [g_0])$ and prove the existence of the above problem provided that $Y_{4,2}(M, [g_0]) < Y_{4,2}(\mathbb{S}^n, [g_{\mathbb{S}^n}])$ by a flow method. This strict inequality is proved if (M, g) is not conformally equivalent to the round sphere.

This is a joint work with Wei Wei and Guofang Wang.